INTEGRAL EQUATIONS FOR SOME INVERSE PROBLEMS OF RADIATIVE HEAT EXCHANGE

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We consider the derivation and analysis of integral equations for some inverse problems of radiative heat exchange.

The majority of the papers in the field of inverse problems of heat exchange have been devoted to the methods and applications of inverse problems of heat conduction [1-3]. From the mathematical point of view, these problems can be treated as inverse problems for the differential equation of heat conduction. The treatment of radiative heat exchange processes is normally based on integral and integrodifferential equations [4, 5]. Problems leading to inverse problems of radiative heat exchange (IPRHE) occur primarily in the design of various technological devices and in the interpretation of experiments on thermal processes where the transport of heat by radiation is dominant. Inverse problems for integral equations of radiative transport in different fields were considered in [6-9] and in [4, 10-12] integral equations were considered for particular formulations of inverse radiative heat-exchange problems in a system of diffusely radiating and reflecting bodies distributed in a diathermal medium. In continuation of these studies, we consider in the present paper the integral equations for a wider class of IPRHE. Inverse problems of heat exchange are in many cases incorrectly posed problems [1-3]. This is also true in IPRHE [4, 10], and, as will be seen below, different equations can be used in the solution of a particular inverse problem of radiative heat exchange. Hence the problem is to choose equations such that minimum mathematical manipulation of the initial data is required to solve the problem.

In heat engineering and thermal physics research, and also in measurements, we most often consider fluxes of effective E_{eff}, resultant E_r, and incident E_{inc} radiation. We write down several known results of radiative heat exchange which will be necessary in the discussion below: [4, 5]:

$$E_{eff} = E_c + RE_{inc} = E_r + E_{inc}, \tag{1}$$

$$E_r = E_c - (1-R)E_{inc} = (E_c - (1-R)E_{eff})/R, \tag{2}$$

$$E_{inc}(M) = \int_F E_{eff}(N) K(M, N)\, dF_N. \tag{3}$$

The division of radiative heat exchange problems into direct and inverse problems will be done in correspondence with the generally accepted principle [1-3] based on the assignment of the quantity to be found to either the characteristics of cause or effect in the process being considered. The principle characteristics in radiative heat exchange problems are the temperature and optical properties of the surfaces, and the geometry of the system of heat-exchanging bodies (the kernel of Eq. (3)). E_c is determined directly in terms of these characteristics and can be considered as a cause characteristic. All of the other radiation fluxes (E_{eff}, E_{inc}, E_r) are effect characteristics. From this point of view, an inverse radiative heat exchange problem consisting of the determination of the temperature according to a specified resultant radiation flux [4, 12] is a particular case of the set of inverse problems considered here. Similar inverse and direct radiative heat exchange problems are related to one another and can easily be united into a single generalized problem [12].

The derivation of the integral equations for inverse problems of radiative heat exchange is based on transformations of the equations for the corresponding direct (fundamental) radiative heat exchange problem. They have the following forms [4, 5]:

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$$E_{\text{eff}}(M) - R(M)\int_F E_{\text{eff}}(N)K(M,N)\,dF_N = E_c(M),\ M \in F; \tag{4}$$

$$E_{\text{inc}}(M) - \int_F R(N)E_{\text{inc}}(N)K(M,N)\,dF_N = \int_F E_c(N)K(M,N)\,dF_N,\ M \in F; \tag{5}$$

$$E_r(M)/\varepsilon(M) - \int_F (1-\varepsilon(N))/\varepsilon(N)\,E_r(N)K(M,N)\,dF_N = E_c(M)/\varepsilon(M) - \int_F E_c(N)/\varepsilon(N)\,K(M,N)\,dF_N,\quad M \in F. \tag{6}$$

TABLE 1. Integral Equations for IPRHE of Group A

Specified	To be found	Equation	
E_{eff}	E_c	$E_c(M) = E_{\text{eff}}(M) - R(M)\int_F E_{\text{eff}}(N)K(M,N)\,dF_N,\ M \in F$	(7)
E_{inc}	E_{eff}	$\int_F E_{\text{eff}}(N)K(M,N)\,dF_N = E_{\text{inc}}(M),\ M \in F;$	(8)
	E_c	$E_c(M) = E_{\text{eff}}(M) - R(M)E_{\text{inc}}(M)$	
E_r	E_{eff}	$E_{\text{eff}}(M) - \int_F E_{\text{eff}}(N)K(M,N)\,dF_N = E_r(M),\ M \in F;$	(9)
	E_c	$E_c(M) = (1 - R(M))E_{\text{eff}}(M) + R(M)E_r(M)$	

Equations (4)-(6) are Fredholm integral equations of the second kind. The theory of this type of equation, as well as methods of solution have been worked out extensively [4, 5, 13].

An IPRHE consists of the determination of certain cause characteristics in terms of other known cause characteristics and known or specified effect characteristics. The quantities to be found in an inverse problem can be any of the cause characteristics or combinations of these characteristics. Here we consider only IPRHE which can be formulated in terms of integral equations of a known type (class) [13]. They include problems of determining the temperature (temperature IPRHE) or optical properties (optical IPRHE) [4, 14]. For the solution of geometrical (configurational) IPRHE, variational methods are necessary [4].

In practice it is convenient to divide the various inverse problems into the following three groups, depending on the method of specifying the initial information at the surface of the radiating system.

A. One of the radiation fluxes (besides E_c) is specified on the entire surface F.

B. Different radiation fluxes are specified on two parts F_1 and F_2 of the surface F.

C. Two different radiation fluxes are known on one of the parts of the surface F_1.

We consider the procedure of deriving the integral equations for the case of temperature IPRHE, which are of the most interest in practice [10, 11, 14].

If any of the radiation fluxes pertaining to effect characteristics is specified on the entire surface (a problem of group A), the integral equation for E_c (and thus the temperature) can in principle be obtained by substituting the known quantities into the left hand sides of equations (4)-(6). Analysis shows that the problems of this type can be solved more effectively in two steps. In the first step we determine E_{eff} from the corresponding integral equation, and in the second we determine E_c from (1) or (2) in terms of the two known radiation fluxes. Then all inverse problems of this group, for all possible combinations of specified radiation fluxes, can be described (for known E_{eff}, E_{inc}, or E_r) in terms of Fredholm integral equation of the first or second kind (see Table 1). The utility of this approach is that the specified function is used directly in the solution of the integral equation, without additional transformations and thus the error in the right hand side will be a minimum. This is an important condition in finding regular solutions of integral equations; particularly Fredholm equations of the first kind [13].

TABLE 2. Systems of Integral Equations for IPRHE of Group B

Specified F_1	Specified F_2	To be found F_1	To be found F_2	System of equation	
E_c	E_{eff}	E_{eff}	E_c	$E_{eff} - R\int_{F_1} E_{eff}K_1dF = E_c + R\int_{F_2} E_{eff}K_1dF,\ M \in F_1;$ $E_c + R\int_{F_1} E_{eff}K_2dF = E_{eff} - R\int_{F_2} E_{eff}K_2dF,\ M \in F_2$	(10)
E_c	E_{inc}	E_{inc}	E_c	$E_{eff} - R\int_{F} E_{eff}KdF = E_c,\ M \in F_1;$ $\int_{F} E_{eff}KdF = E_{inc},\ M \in F_2;$	(11)
E_{inc}	E_{eff}	E_{eff}	E_{inc}	$E_{inc} = (E_{eff} - E_c)/R,\ M \in F_1;\ E_c = E_{eff} - RE_{inc},\ M \in F_2;$ $\int_{F_1} E_{eff}K_1dF = E_{inc} - \int_{F_2} E_{eff}K_1dF,\ M \in F_1;$ $E_{inc} = \int_{F_1} E_{eff}K_2dF + \int_{F_2} E_{eff}K_2dF,\ M \in F_2;$	(12)
		(E_c)	(E_c)	$E_c = E_{eff} - RE_{inc}$	
E_r	E_{eff}	E_{eff}	E_r	$E_{eff} - \int_{F_1} E_{eff}K_1dF = E_r + \int_{F_2} E_{eff}K_1dF,\ M \in F_1;$ $E_r = E_{eff} - \int_{F_1} E_{eff}K_2dF - \int_{F_2} E_{eff}K_2dF,\ M \in F_2;$	(13)
		(E_c)	(E_c)	$E_c = (1 - R)\,E_{eff} + RE_r$	
E_{inc}	E_r	E_r	E_{inc}	$\int_{F} E_{eff}K_1dF = E_{inc},\ M \in F_1;$ $E_{eff} - \int_{F} E_{eff}K_2dF = E_r,\ M \in F_2;$	(14)
		(E_c)	(E_c)	$E_r = E_{eff} - E_{inc}\ \ M \in F_1;\ \ E_{inc} = E_{eff} - E_r,\ \ M \in F_2;$ $E_c = E_{eff} - RE_{inc}$	

For inverse problems where different combinations of radiation fluxes are known on two parts of the surface (problems of group B), the integral equation can also be derived from equations (4)-(6), (relating the two specified fluxes) as a system of two equations written for each part of the surface F_1 and F_2. However, the resulting system of integral equations will have a simpler structure if we take E_{eff} as an intermediate quantity to be found, as in the analysis of problems of group A. Then all inverse problems of group B can be represented as five systems of equations, as shown in Table 2.

Each of the equations of the system is one of the equations (3), (4) or (9) transformed according to whether the specified quantity belongs to F_1 or F_2. Analysis of these systems reveals certain characteristic traits. If the flux density of incident radiation is specified on a part of the surface, then one of the equations of the system is a Fredholm integral equation of the first kind (systems (11), (12), and (14)). If the flux density of effective radiation is specified on a part of the surface F_2 (systems (10), (12), and (13)) then the equations are solved successively in two steps: first the unknown radiation fluxes are determined independently from the equations for the part of the surface F_1; then all of the radiation fluxes are found on F_2. An iteration method must be used to solve this system, which consists of equations of the first and second kinds.

With the help of relations (1) and (2), any problem of group C can always be reduced to a problem in which both the flux density of effective radiation and the flux density of incident radiation are specified on F_1. One encounters similar problems in the determination of specified heat flux fields with radiative heating devices [10]. In this case the determination of the unknown quantities on F_2 is based on the solution of a system of equations which are simpler in structure than those for other cases:

$$\int_{F_2} E_{\text{eff}}(N)\,K(M,\ N)\,dF_N = E_{\text{inc}}\ (M) - \int_{F_1} E_{\text{eff}}(N)\,K(M,\ N)\,dF_N,\ M \in F_1;$$

$$E_{\text{inc}}\ (M) - \int_{F_2} E_{\text{eff}}(N)\,K(M,\ N)\,dF_N = \int_{F_1} E_{\text{eff}}(N)\,K(M,\ N)\,dF_N,\ M \in F_2.$$

The first of these equations is a Fredholm equation of the first kind and can be solved independently of the second. Substitution of the obtained flux density of effective radiation on F_2 into the second equation yields at once the flux density of incident ratiation on F_2.

For the inverse problems of radiative heat exchange considered above it was required to determine the temperature of a surface (for known optical property). We briefly consider now the case where the optical property ($\varepsilon = 1 - R$) is unknown, while the temperature is known.

For inverse problems of group A, the determination of ε reduces to the solution of the equations shown in Table 1. Then using the Stefan—Boltzmann law for E_c, the problem reduces to the solution of a linear equation for ε. For inverse problems of group B and C, when any of the radiation fluxes except E_c are specified on parts of the surface, the determination of the optical properties also reduces to the solution of the systems of equations considered above and then calculation from these equations of a linear equation for ε or R.

A qualitative analysis of the integral equations considered here shows that the solution of Fredholm integral equations of the second kind, figuring in direct and inverse radiative heat exchange problems, does not cause particular difficulties. But the choice of an effective method of solving Fredholm equations of the first kind, and systems of these equations describing the inverse problems considered above, requires additional study.

NOTATION

$E_c = E_c(M)$, $E_{eff} = E_{eff}(M)$, $E_{inc} = E_{inc}(M)$, $E_r = E_r(M)$, respectively, flux densities of the characteristic, effective incident, and resultant radiation $\varepsilon = \varepsilon(M)$, $R = R(M)$, integral emissivity and reflectivity of the surface in the neighborhood of the point M; F, surface area of the system of bodies; F_1, F_2, parts of the surface F; K(M, N), kernel of the integral equation, completely determined by the geometry of the system of bodies; $K_1(M, N) = K(M \in F_1, N)$; $K_2(M, N) = K(M \in F_2, N)$.

LITERATURE CITED

1. O. M. Alifanov, Identification of Heat Exchange Processes in Aircraft [in Russian], Mashinostroenie, Moscow (1979).
2. L. A. Kozdoba and P. G. Krukovskii, Methods of Solution of Inverse Problems of Heat Transport [in Russian], Naukova Dumka, Kiev (1982).
3. D. F. Simbirskii, Temperature Diagnostics of Engines [in Russian], Tekhnika, Kiev (1976).
4. N. A. Rubtsov, Radiative Heat Exchange [in Russian], Novosibirsk State Univ. (1977).
5. V. N. Adrianov, Foundations of Radiative and Complex Heat Exchange [in Russian], Énergiya, Moscow (1972).
6. A. V. Goncharskii, A. M. Cherepashchuk, and A. G. Yagola, Numerical Methods of Solving Inverse Problems in Astrophysics [in Russian], Nauka, Moscow (1978).
7. G. A. Kolobova, "Some questions on the diagnostics of low-temperature plasmas," Collection of Scientific Papers "Radiative Heat Exchange" [in Russian], Inst. Thermal Physics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1977), pp. 42-50.
8. A. Ya. Kazakov, "Inverse problem of radiative transport with a spherical scattering indicatrix," Vestn. Leningr. Fiz.-Gos. Univ., Fiz.-Khim., Issue 2, No. 10, 88-91 (1982).
9. V. V. Pikalov and N. G. Preobrazhenskii, "Calculation of the local equilibrium temperature field of a medium from its radiation," Inzh.-Fiz. Zh., 33, No. 6, 1042-1046 (1977).
10. N. G. Zamula and V. M. Yudin, "Some inverse problems of heat conduction and radiation," Prikl. Mekh., 4, Issue 1, 111-117 (1968).
11. N. A. Rubtsov, Radiative Heat Exchange in a Layered Medium [in Russian], Nauka, Novosibirsk (1984).
12. Yu. A. Surinov, "Solution of the integral equations of radiation for generalized three-dimensional problems," Izv. Vyssh. Uchebn. Zaved., Fiz., No. 3, 78-87 (1964).
13. A. F. Verlan' and V. S. Sizikov, Methods of Solution of Integral Equations (with Computer Programs) [in Russian], Naukova Dumka, Kiev (1978).

14. Yu. K. Lingart and V. A. Petrov, "Approximation method of calculating the true temperature from infrared pyrometer readings in the presence of a radiation background," Teplofiz. Vys. Temp., 16, No. 5, 1046-1053 (1978).

SOLUTION OF INVERSE PROBLEMS OF RADIATIVE TRANSPORT BY SOOT PARTICLES OF COMPLEX SHAPES

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We work out a method of determining the effective parameters of soot particles from their radiation characteristics. The method is based on the measurements of the spectral transmission coefficient in the infrared region of the spectrum.

It is known that the radiative properties of hydrocarbon flames are determined mainly by a polydisperse system of soot particles with complicated shapes and with a wide spectrum of sizes (from 0.02 to 5 μm) [1]. The current methods of calculating the radiative characteristics are based on the representation of the soot particles as spheres or ellipsoids of the same volume as the actual aggregate. Also the effect of the sizes, shapes, and orientations of the particles in space on the radiative characteristics of the flame are not taken into account. The data from optical measurements cannot be used to establish the concentration of soot in the combustion products because of the unsatisfactory agreement with measurements by contact methods [1, 2].

The refinement and development of optical diagnostics of hydrocarbon propellant flames in the presence of a dust of soot particles requires 1) quantitative relations between the effective parameters of the soot particles which determine their radiative characteristics, and the sizes, shapes, and orientations of the particles in space, and 2) solution of the inverse problem of radiative transport by particles of complicated shapes, i.e., the determination of these parameters from measurements of attenuation or angular scattering upon probing the medium by sources of radiative energy.

The solution of the first problem reduces to the choice of an optical model of the soot particles which would give the dependence of the radiative spectral characteristics of the particles on their sizes, shapes, and orientations in space on the basis of the Mie theory of the interaction of a flux of radiation with a spherical particle.

In [3] this problem was studied analytically and an optical model of the soot particles was worked out for the interpretation of attenuation measurements of the radiation flux by the soot particles. The model uses the following assumptions:

1) the soot particles are represented as clusters of elementary spheres of diameter d_0, and the number of spheres and their relative positions determines the size and shape of the aggregates;

2) the spectral attenuation coefficient of the soot particles $k_\lambda(D^*)$ is determined by the effective size D^*, which is the diameter of a circle with an area equal to the cross-sectional area of the aggregate $F_i = \pi D_i^{*2}/4 = \pi d_0^2 m_i/4$, where m_i is the number of elementary particles whose areas projected onto a plane perpendicular to the direction of the flux makes up the area of the irradiated surface;

3) the distribution function of the parameter m for soot particles consisting of n_j elementary particles obeys a normal distribution;

4) the soot particles are oriented randomly in space.

The quantitative relation between the effective parameters of the soot particles and their mass concentrations by size is given in terms of equations which were obtained with the

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